Search for Yield

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Abstract
We present a model of the relationship between real interest rates, credit spreads, and the structure and risk of the banking system. Banks intermediate between entrepreneurs and investors, and can monitor entrepreneurs’ projects. We characterize the equilibrium for a fixed aggregate supply of savings, showing that safer entrepreneurs will be funded by nonmonitoring banks and riskier entrepreneurs by monitoring banks. We show that an increase in savings reduces interest rates and spreads, increases the relative size of the nonmonitoring banking system and the probability of failure of monitoring banks. We also show that the dynamic version of the model exhibits endogenous boom and bust cycles, and rationalizes the existence of countercyclical risk premia and the connection between low interest rates, credit spreads, and the buildup of risks during booms.

JEL Classification: G21, G23, E44

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1 Introduction

The connection between interest rates and financial stability has been the subject of extensive discussions and significant amount of (mostly empirical) research. This paper contributes to this literature by constructing a theoretical model of the relationship between real interest rates, credit spreads, and the structure and risk of the banking system. It thus provides a framework to understand how an increase in savings (a “global savings glut”) that reduces the level of long-term real interest rates, noted by Bernanke (2005), can generate incentives to “search for yield” and increases of risk-taking that can lead to financial instability, as noted by Rajan (2005) and Summers (2014).

The model shows that an increase in savings reduces interest rates and credit spreads, increases the relative size of the nonmonitoring banking system, and increases the probability of failure of monitoring banks. In terms of interpretation, monitoring banks may be associated with traditional banks that originate-to-hold, while nonmonitoring banks may be related with either direct market finance, with institutions that originate-to-distribute, or with shadow banks.\footnote{This use of the term shadow banks follows the Financial Stability Board (2014): “The shadow banking system can broadly be described as credit intermediation involving entities and activities outside of the regular banking system.” They note that some authorities and market participants prefer to use other terms such as “market-based financing.”}

The model can rationalize the existence endogenous boom and bust cycles. The accumulation of savings in a boom leads to a reduction in interest rates and spreads, which increases risk-taking that eventually materializes in a bust. The bust reduces savings and increases interest rates and spreads, starting again the process of wealth accumulation that leads to a boom. The model also yields a number of empirically relevant results such as the procyclical nature of risk-taking in the banking system, the procyclical size of the nonmonitoring banking system, and the existence of countercyclical risk premia. These findings contribute to our understanding of the role of financial factors in economic fluctuations.

The paper starts with a simple partial equilibrium model of bank lending with three types of risk-neutral agents: entrepreneurs, investors, and a bank. Entrepreneurs seek bank
finance for their risky investment projects. The bank, in turn, needs to raise funds from a set of (uninsured) investors. Following Holmström and Tirole (1997), the bank can decide the monitoring intensity of entrepreneurs’ projects at a cost, which reduces their probability of failure. Monitoring is not contractible, so there is a moral hazard problem. We characterize the optimal contract between the bank and the investors, showing that there are circumstances in which the bank chooses not to monitor entrepreneurs and others in which it chooses to do so.

The partial equilibrium results show that which case obtains depends on the spread between the bank’s lending rate and the expected return required by investors, which under risk-neutrality equals the safe rate. In particular, a reduction in this spread reduces monitoring, and makes it more likely that the bank will not find it optimal to monitor entrepreneurs.

To endogenize interest rates and credit spreads we embed our model of bank finance into a static general equilibrium setup. In such setup, a large set of heterogeneous entrepreneurs (that differ in their observable risk type) seek funding for their investment projects from a competitive banking sector. Assuming a downward-sloping demand for investment for each type,\(^2\) we characterize the equilibrium for a fixed aggregate supply of savings, showing that safer entrepreneurs will be funded by nonmonitoring banks and riskier entrepreneurs by monitoring banks.

We then analyze the effects of an exogenous increase in the aggregate supply of savings, showing that it will lead to a reduction in interest rates and credit spreads, an increase in investment and bank lending to all types of entrepreneurs, an expansion of the relative size of the nonmonitoring banking system, and a reduction in the monitoring intensity and, hence, an increase in the probability of failure of monitoring banks. These results provide a consistent explanation of a number of stylized facts of the period preceding the 2007-2009 financial crisis; see, for example, Brunnermeier (2009).

Interestingly, we show that the equilibrium of the model is constrained inefficient, in

\(^2\)We provide a microfoundation for this assumption in terms of either the demand of a set of final good producers that use entrepreneurs’ output as an intermediate input, or from the demand of a representative consumer with a utility function over the continuum of goods produced by entrepreneurs of different types.
the sense that a social planner subject to the same moral hazard problem as the banks could improve upon the equilibrium allocation. In particular, the social planner would shift investments from riskier to safer entrepreneurs. This will increase credit spreads and hence monitoring incentives, so banks will be safer.

Although we focus on the effects of an exogenous increase in the supply of savings, the same effects obtain when there is an exogenous decrease in the demand for investment. Thus, the model provides an explanation of the way in which factors leading to a reduction in the equilibrium real interest rate, as those noted by Summers (2014), can be linked to an increase in financial instability.

Finally, we consider a dynamic version of the static general equilibrium setup in which investors are infinitely lived and the aggregate supply of savings is endogenous. Specifically, the supply of savings at any date is the outcome of investors’ consumption and savings decisions at the previous date together with the realization of a systematic risk factor that affects the return of entrepreneurs’ projects. For good realizations of the risk factor, aggregate savings will accumulate leading to lower interest rates and spreads, which translate into higher risk-taking and a fragile financial system. In this situation, the economy is especially vulnerable to a bad realization of the risk factor, which can lead to a crisis. The occurrence of a crisis results in a reduction in aggregate savings leading to higher interest rates and spreads, which translate into lower risk-taking and a safer financial system. Then savings will grow, restarting the process that produces another boom. In this manner, the model generates endogenous boom and bust cycles.

The dynamic model yields other relevant and potentially testable results. First, interest rates and credit spreads are countercyclical. Second, during booms the safe rate may be below investors’ subjective discount rate. Third, the nonmonitoring banking system is highly procyclical. Fourth, even though investors are risk-neutral, risky assets have positive risk premia. Fifth, even though investors’ preferences do not change over time, such risk premia are countercyclical.

The brief review of the literature that follows discusses the relation to previous studies and the evidence on some of these predictions.
Literature review  This paper is linked to different strands of the (theoretical and empirical) literature on the relationship between interest rates, financial frictions, the structure of the financial system, and the business cycle.

Our interest in the effects of financial frictions on economic activity relates to numerous studies following the seminal papers of Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1996), and Kiyotaki and Moore (1997).3 We have chosen to introduce these frictions using the moral hazard setup of Holmström and Tirole (1997). We depart from their model by focussing exclusively on banks’ moral hazard problem, endogeneizing the return structure that entrepreneurial projects offer in a competitive setup, and introducing heterogeneity in the ex-ante risk profile of entrepreneurs instead of in their net worth. In their characterization of equilibrium, entrepreneurs with low net worth borrow from monitoring banks while those with high net worth are directly funded by the market. In contrast, in our setup riskier entrepreneurs borrow from monitoring banks while safer entrepreneurs borrow from nonmonitoring banks.

Most papers that analyze the role of financial intermediaries in economic fluctuations focus on leverage; see, for example, Gertler and Kiyotaki (2010), Repullo and Suarez (2013), and Adrian and Shin (2014). We depart from this literature by considering a model in which banks have no equity capital.4

Our work is related to a large volume of research spurred after the 2007-2009 financial crisis. On the one hand, we provide a theoretical framework that links a savings glut with the level of interest rates and the increases in risk-taking noted by Rajan (2005) and Summers (2014), among many others. On the other, we obtain predictions regarding the behavior of interest rates and spreads, risk premia, and the structure and risk of the banking system in line with recent empirical findings. For example, Lopez-Salido, Stein, and Zakrajšek (2016) show that the widening of credit spreads following a period of low spreads is closely tied to

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3See Quadrini (2011) and Brunnermeier, Eisenbach, and Sannikov (2013) for surveys of macroeconomic models with financial frictions, and Adrian, Colla, and Shin (2013) for a review of the performance of these models in explaining key features of the 2007-2009 financial crisis.

4A follow up paper, Martinez-Miera and Repullo (2016), introduces capital to discuss the effects of different types of capital requirements.
a contraction in economic activity.\footnote{They interpret this result in behavioral terms (a change in “credit market sentiment”), whereas our story does not rely on changes in investors’ preferences.} Our results on risk premia are also in line with Gilchrist and Zakrajšek (2012), who find a negative relationship between risk premia and economic activity, and Muir (2016), who finds that risk premia increase substantially in financial crises. Finally, our results on the procyclicality of the nonmonitoring banking system are consistent with the evidence in Pozsar, Adrian, Ashcraft, and Boesky (2012).

Our focus on how endogenously determined interest rates affect banks’ decisions in a general equilibrium setting is related to Boissay, Collard, and Smets (2016). They analyze a model with an interbank market where lower interest rates make riskier banks more prone to borrow from safer banks. Their paper, like ours, generates endogenous boom and bust cycles which are driven by banks’ strategic responses to changes in interest rates. Our papers depart in that we abstract from the interbank market, and consider instead the effect of interest rates on banks’ monitoring decisions.\footnote{It should be noted that, as in Brunnermeier and Sannikov (2014) or He and Krishnamurthy (2012), we do not analyze a linearized version of the model but instead solve the full equilibrium dynamics.}

Our focus on the connection between a savings glut and financial stability is related to Bolton, Santos, and Scheinkman (2016). They analyze the effects of a savings glut on the incentives to originate high quality assets by possibly informed intermediaries. In contrast to our paper, one of their main building blocks is the presence of cash-in-the-market pricing, taking into account liquidity and leverage considerations.

Many empirical papers analyzing the link between interest rates and banks’ risk-taking focus on monetary policy. Although we have a real model without nominal frictions, some of this evidence is also in line with our predictions; see, for example, Jimenez, Ongena, Peydro, and Saurina (2014), Altunbas, Gambacorta, and Marques-Ibanez (2014), Dell’Ariccia, Laeven, and Suarez (2016), and Ioannidou, Ongena and Peydro (2015).

**Structure of the paper**  Section 2 presents the partial equilibrium model of bank lending under moral hazard. Section 3 embeds the partial equilibrium model into a general equilibrium setup, characterizing the equilibrium for a fixed aggregate supply of savings, analyzing
the effects of an increase in the supply savings, and showing the constrained inefficiency of the competitive equilibrium allocation. Section 4 analyzes the dynamic version of the model that generates endogenous booms and busts, and Section 5 presents some concluding remarks. Appendix A considers an extension of the static model in which investors are risk-averse, which provides a way to empirically distinguish a savings glut from a reduction in investors’ risk appetite. Appendix B contains the proofs of the analytical results of the paper.

2 Partial Equilibrium

Consider an economy with two dates \((t = 0, 1)\), a large set of penniless entrepreneurs, a large set of risk-neutral investors, and a single risk-neutral bank. Entrepreneurs have investment projects that require external finance, which can only come from the bank. The bank, in turn, needs to raise funds from the investors, which are characterized by an infinitely elastic supply of funds at an expected return equal to \(R_0\).

Each entrepreneur has a project that requires a unit investment at \(t = 0\) and yields a stochastic return \(\tilde{R}\) at \(t = 1\) given by

\[
\tilde{R} = \begin{cases} 
    R, & \text{with probability } 1 - p + m, \\
    0, & \text{with probability } p - m,
\end{cases}
\]

where \(R > 0\) and \(p \in (0, 1)\) are constant parameters, and \(m \in [0, p]\) is a variable that captures the bank’s monitoring intensity. Monitoring increases the probability of getting the high return \(R\), but entails a cost \(c(m)\). We assume that monitoring is not observed by the investors, so there is a moral hazard problem.

The monitoring cost function \(c(m)\) satisfies \(c(0) = c'(0) = 0\), \(c'(m) \geq 0\), \(c''(m) > 0\), and \(c'''(m) \geq 0\). A special case that satisfies these assumptions and will be used for our numerical results is the quadratic function

\[
c(m) = \frac{\gamma}{2} m^2,
\]

where \(\gamma > 0\).
The bank can only fund a limited set of projects, taken to be just one for simplicity. Thus, entrepreneurs will be on the short side of the market and so they will only be able to borrow at the rate $R$ that leaves them no surplus. The bank will raise a unit of funds from investors at a rate $B$, and given the loan rate $R$ it will choose a monitoring intensity $m \in [0, p]$.

An *optimal contract* between the bank and the investors is a pair $(B^*, m^*)$ that solves

$$\max_{(B,m)} [(1 - p + m)(R - B) - c(m)]$$

subject to the bank’s incentive compatibility constraint

$$m^* = \arg \max_m [(1 - p + m)(R - B^*) - c(m)],$$

the bank’s participation constraint

$$(1 - p + m^*)(R - B^*) - c(m^*) \geq 0,$$

and investors’ participation constraint

$$(1 - p + m^*)B^* = R_0.$$  

The incentive compatibility constraint (4) characterizes the bank’s choice of monitoring $m^*$ given the borrowing rate $B^*$ and the loan rate $R$. The participation constraints (5) and (6) ensure that the bank makes nonnegative profits, net of the monitoring cost, and that investors get the required expected return on their investment.

An interior solution to (4) is characterized by the first-order condition

$$R - B^* - c'(m^*) = 0.$$  

Solving for $B^*$ in the participation constraint (6), substituting it into the first-order condition (7), and rearranging gives the equation

$$c'(m^*) + \frac{R_0}{1 - p + m^*} = R.$$
Since we have assumed $c''(m) \geq 0$, the function in left-hand side of (8) is convex in $m$. Let $\bar{R}$ denote the minimum value of this function in the feasible range $[0, p]$, that is

$$\bar{R} = \min_{m \in [0, p]} \left( c'(m) + \frac{R_0}{1 - p + m} \right).$$

(9)

The following result shows the condition under which bank finance is feasible and characterizes the corresponding optimal contract between the bank and the investors.

**Proposition 1** Bank finance is feasible if $R \geq \bar{R}$, in which case the optimal contract between the bank and the investors is given by

$$m^* = \max \left\{ m \in [0, p] \mid c'(m) + \frac{R_0}{1 - p + m} \leq \bar{R} \right\} \text{ and } B^* = \frac{R_0}{1 - p + m^*}.$$  

(10)

Proposition 1 states that bank finance is feasible if the lending rate $R$ is greater than or equal to the minimum value $\bar{R}$ defined by (9), in which case the optimal contract is characterized by the highest value of $m$ that satisfies

$$c'(m) + \frac{R_0}{1 - p + m} \leq \bar{R}.$$

Monitoring in the optimal contract may be at the corner with zero monitoring $m^* = 0$, at the corner with full monitoring $m^* = p$, or it may be interior $m^* \in (0, p)$. Since monitoring is costly and it is not observed by investors, the bank will never monitor the entrepreneur when it is going to sell the loan, because it will get no compensation for its monitoring. Assuming that the bank sells the loan when it is indifferent between keeping it and selling it, we may then associate the case $m^* = 0$ with either direct market finance or with institutions that originate-to-distribute, and the case $m^* > 0$ with traditional banks that originate-to-hold.
Figure 1. Characterization of the optimal contract

Panel A shows a case in which the optimal contract may entail zero monitoring, and Panel B a case where the optimal contract always has positive monitoring.

Figure 1 illustrates the two modes of finance for the quadratic monitoring cost function. Panel A shows a case where the slope of the function in left-hand side of (8) is nonnegative at the origin, in which case the optimal contract may entail $m^* = 0$ (for $R = \bar{R}$). Panel B shows a case where the slope of this function is negative at the origin, in which case the optimal contract always entails $m^* > 0$.

We next derive some interesting comparative static results on the optimal contract, assuming that it involves an interior level of monitoring.

**Proposition 2** If $R > \bar{R}$, monitoring in the optimal contract $m^*$ is decreasing in $R_0$ and increasing in $R$.

Thus, a reduction in the credit spread $R - R_0$ due to either an increase in the funding cost $R_0$ or a decrease in the loan rate $R$ reduces optimal monitoring, thereby increasing the bank’s portfolio risk. For sufficiently low spreads, the bank may find it optimal to choose zero monitoring, switching from originate-to-hold to originate-to-distribute. Figure 1 illustrates the second result in Proposition 2: whenever bank finance is feasible, a reduction in the loan rate from $R$ to $\bar{R}$ reduces monitoring from $m^*$ to $m^*$.

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7Which case obtains depends on the sign of $c''(0) - R_0/(1-p)^2$. 
3 General Equilibrium

This section embeds our partial equilibrium model of bank finance into a general equilibrium model with a fixed aggregate supply of savings in which all interest rates are endogenous. The model has a continuum of heterogeneous entrepreneurs that differ in their observable risk type. We characterize the competitive equilibrium and show that safer entrepreneurs will borrow from nonmonitoring banks while riskier entrepreneurs will borrow from monitoring banks. We then analyze the effects of an exogenous increase in the supply of savings, showing that it will lead to a reduction in interest rates and credit spreads, and an increase in the risk of the banking system. Finally, we prove that the equilibrium allocation is constrained inefficient, in the sense that a social planner subject to the same moral hazard problem as the banks would shift investments from riskier to safer entrepreneurs.

Consider an economy with two dates \((t = 0, 1)\) and a large set of penniless entrepreneurs with observable types \(p \in [0, 1]\). Entrepreneurs have investment projects that require external finance, which can only come from banks. Banks are risk-neutral agents that specialize in lending to specific types of entrepreneurs. To simplify the presentation, we will assume that for each type \(p\) there is a single bank that only lends to entrepreneurs of this type.\(^8\) Banks, in turn, need to raise funds from a set of investors, which are characterized by a fixed aggregate supply of savings \(w\).

Each entrepreneur of type \(p\) has a project that requires a unit investment at \(t = 0\) and yields a stochastic return \(\tilde{R}_p\) at \(t = 1\) given by

\[
\tilde{R}_p = \begin{cases} 
R_p, & \text{with probability } 1 - p + m, \\
0, & \text{with probability } p - m,
\end{cases}
\]

where \(m \in [0, p]\) is the monitoring intensity of its bank. As before, monitoring is costly and the monitoring cost \(c(m)\) satisfies our previous assumptions. The returns of the projects of entrepreneurs of each type \(p\) are assumed to be perfectly correlated (but we could have correlation across different types; see Section 4). This implies that the bank’s return per

\(^8\)Without loss of generality, we could have many banks lending to each type of entrepreneur. What might be restrictive is the assumption of banks specializing in lending to only one type of entrepreneurs. However, Repullo and Suarez (2004) show that, for a model with insured deposits, it is optimal for banks to specialize.
unit of loans is identical to the individual project return, which is given by (11).

We assume that the success return \( R_p \) is a decreasing function \( R(x_p) \) of the aggregate investment of entrepreneurs of type \( p \), denoted \( x_p \). Thus, the higher the aggregate investment \( x_p \) the lower the return \( R_p \).

This assumption may be rationalized by assuming that each entrepreneur of type \( p \) produces (in case of success) a unit of an intermediate input sold at a price \( R_p \) to a set of final good producers. Specifically, suppose that for each \( p \) there is a continuum of final good producers with heterogeneous productivity \( \theta_p \), which is distributed according to the density \( g(\theta_p) = a\sigma (\theta_p)^{-(\sigma+1)} \), where \( a > 0 \) and \( \sigma > 1 \).

Each producer can transform a unit of the intermediate input into \( \theta_p \) units of the final good, which is assumed to have a unit price. For any price of the intermediate input \( R_p \), only producers with productivity \( \theta_p \geq R_p \) will operate. Hence, given an aggregate supply of the intermediate input \( x_p \) we must have

\[
\int_{R_p}^{\infty} g(\theta_p) \, d\theta_p = a (R_p)^{-\sigma} = x_p, \tag{12}
\]

which implies

\[
R_p = R(x_p) = \left( \frac{x_p}{a} \right)^{-1/\sigma}. \tag{13}
\]

Notice that \( \sigma \) is the elasticity of the demand for the intermediate input, while \( a \) is a (proportional) demand shifter related to the productivity of the final good producers. This function (with \( a = 1 \)) will be used to derive the numerical results of the paper.

Alternatively, we could introduce a representative consumer with a utility function over the continuum of goods produced by entrepreneurs of types \( p \in [0,1] \). Specifically, assume that

\[
U(q, x) = q + \frac{a\sigma}{\sigma - 1} \int_0^1 \left( \frac{x_p}{a} \right)^{\frac{\sigma-1}{\sigma}} dp, \tag{14}
\]

where \( q \) is the consumption of a composite good, \( x = \{x_p\}_{p \in [0,1]} \), \( a > 0 \) and \( \sigma > 1 \). Maximizing the utility of the representative consumer subject to the budget constraint

\[
q + \int_0^1 R_p x_p \, dp = I
\]

The assumption of firms with heterogeneous productivities follows the approach of Melitz (2003).
gives a first-order condition that also implies (13).

If the bank lending to entrepreneurs of type \( p \) sets a loan rate \( L_p \), then a measure \( x_p \) of these entrepreneurs will enter the market until \( L_p = R(x_p) \). Thus, as in the partial equilibrium setup, entrepreneurs will only be able to borrow at a rate that leaves them no surplus.

Finally, to determine equilibrium loan rates, we assume that the loan market is contestable. Thus, although there is a single bank that lends to each type, the incumbent could be undercut by an entrant if it were profitable for the entrant to do so.

The strategy for the analysis is going to be as follows. First, we characterize the investment allocation corresponding to any given safe rate \( R_0 \), which is derived from the condition that investors must be indifferent between funding banks lending to entrepreneurs of different types. Then we introduce the market clearing condition that equates the aggregate demand for investment to the aggregate supply of savings to determine the equilibrium safe rate \( R_0^* \).

By contestability, a bank lending to entrepreneurs of type \( p = 0 \) sets a rate equal to the return \( R_0 \) of their projects, since at a lower rate it will make negative profits and at a higher rate it will be undercut by another bank. Similarly, banks lending to entrepreneurs of types \( p > 0 \) set the lowest feasible rate, which by Proposition 1 (together with the assumption of perfectly correlated defaults for each type \( p \)) implies

\[
R_p = \min_{m \in [0, p]} \left( c'(m) + \frac{R_0}{1 - p + m} \right). \tag{15}
\]

The assumptions on the monitoring cost function \( c(m) \) imply that we have a corner solution with zero monitoring if and only if

\[
c''(0) - \frac{R_0}{(1 - p)^2} \geq 0,
\]

which gives \( p \leq \hat{p} \), where

\[
\hat{p} = 1 - \sqrt{\frac{R_0}{c''(0)}}. \tag{16}
\]

Thus, banks lending to (safer) entrepreneurs of types \( p \leq \hat{p} \) will not monitor them, and banks lending to (riskier) entrepreneurs of types \( p > \hat{p} \) will monitor them. The intuition for this result is that since monitoring is especially useful for riskier entrepreneurs, they will have an incentive to borrow from monitoring banks, and since monitoring is less useful for
safer entrepreneurs (and useless for those with \( p = 0 \)), they will borrow from nonmonitoring banks. In what follows we will assume that \( R_0 < c''(0) \), so \( \hat{p} \in (0, 1) \).\(^{10}\)

For nonmonitoring banks (those lending to types \( p \leq \hat{p} \)) loan rates are given by

\[
R_p = R_p = \frac{R_0}{1 - p},
\]

where we have used the assumption \( c'(0) = 0 \). This result implies \((1 - p)R_p = R_0\), so the expected return of the banks’ investments equals the funding cost. Thus, profits of nonmonitoring banks will always be zero.

For monitoring banks (those lending to types \( p > \hat{p} \)) loan rates are given by

\[
R_p = R_p = c'(m_p) + \frac{R_0}{1 - p + m_p},
\]

where the monitoring intensity \( m_p \) satisfies the first-order condition\(^{11}\)

\[
c''(m_p) - \frac{R_0}{(1 - p + m_p)^2} = 0.
\]

This result implies

\[(1 - p + m_p)R_p - R_0 - c(m_p) = (1 - p + m_p)c'(m_p) - c(m_p) > (1 - p)c'(m_p) > 0,
\]

where we have used (18) and the fact that \( m_p c'(m_p) > c(m_p) \) by the convexity of the monitoring cost function. Thus, profits of monitoring banks will always be positive.

We can now state the following results.

**Proposition 3** For any given safe rate \( R_0 < c''(0) \), there exists a marginal type \( \hat{p} \in (0, 1) \) given by (16) such that banks lending to entrepreneurs of types \( p \leq \hat{p} \) do not monitor their borrowers, and banks lending to entrepreneurs of types \( p > \hat{p} \) monitor them. Higher types \( p \) exhibit higher spreads \( R_p - R_0 \) and (for \( p > \hat{p} \)) higher monitoring \( m_p \).

\(^{10}\)The model also works with \( R_0 \geq c''(0) \), but in this case monitoring is so profitable that all banks (except the one lending to safe entrepreneurs of type \( p = 0 \)) would monitor their borrowers.

\(^{11}\)Notice that we cannot have a corner solution with \( m_p = p \) since the slope of the function in the right-hand-side of (15), evaluated at \( m_p = p \), satisfies \( c'(p) - R_0 \geq c''(0) - R_0 > 0 \), where we have used \( c''(m) \geq 0 \) and \( R_0 < c''(0) \).
**Proposition 4** Assuming that $R_0 < c''(0)$, an increase in $R_0$ leads to a reduction in the marginal type $\hat{p}$, an increase in interest rates $R_p$ and credit spreads $R_p - R_0$, and (for $p > \hat{p}$) an increase in monitoring $m_p$.

We are now ready to define an equilibrium, which requires to specify the investment $x_p$ of the different types of entrepreneurs, and hence the rates $R_p = R(x_p)$ at which they will borrow. By our previous results, both are a function of the equilibrium safe rate $R_0^*$.

Formally, a *competitive equilibrium* is an investment allocation $\{x^*_p\}_{p \in [0,1]}$ and corresponding loan interest rates $R^*_p = R(x^*_p)$ such that loan rates satisfy

$$R^*_p = R^*_p = \min_{m \in [0,p]} \left( c'(m) + \frac{R^*_0}{1 - p + m} \right), \text{ for all } p \in [0,1],$$

and the market clears

$$\int_0^1 x^*_p \, dp = w.$$  \hfill (21)

Condition (20) follows from the assumption that the loan market is contestable, so equilibrium loan rates will be at the lowest feasible level $R^*_p$ implied by the equilibrium safe rate $R_0^*$. Condition (21) ensures that the aggregate demand for investment is equal to the aggregate supply of savings $w$. Notice that the investors’ participation constraint ensures that they all get the same expected return $R_0^*$, regardless of the type of bank they fund.

### 3.1 An increase in the supply of savings

To analyze the effects of an exogenous increase in the aggregate supply of savings $w$ notice that the market clearing condition (21) may be written as

$$F(R_0^*) = \int_0^1 R^{-1}(R^*_p) \, dp = w,$$  \hfill (22)

where $x^*_p = R^{-1}(R^*_p)$ is the inverse function of $R^*_p = R(x^*_p)$. Since we have assumed $R'(x_p) < 0$, and $R^*_p$ is increasing in $R_0^*$ by Proposition 4, we have $F'(R_0^*) < 0$, which implies

$$\frac{dR_0^*}{dw} = \frac{1}{F'(R_0^*)} < 0.$$  

Hence, an increase in the supply of savings $w$ leads to a decrease in the safe rate $R_0^*$ and consequently in the rates $R^*_p$ charged to all entrepreneurs. This, in turn, implies a higher investment $x^*_p$ for all types $p$. 

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Since the equilibrium marginal type
\[ p^* = 1 - \sqrt{\frac{R_0^*}{\sigma_p(0)}} \]
is decreasing in the equilibrium safe rate \( R_0^* \), the nonmonitoring region \([0, p^*]\) will be larger. Moreover, by Proposition 4 the decrease in \( R_0^* \) will reduce the monitoring intensity \( m_p^* \) of monitoring banks, so they will become riskier.

We can summarize these results as follows.

**Proposition 5** An increase in the aggregate supply of savings \( w \) leads to

1. A reduction in the safe rate \( R_0^* \) and in the loan rates \( R_p^* \) of all types of entrepreneurs.
2. An increase in investment \( x_p^* \) and hence in bank lending to all types of entrepreneurs.
3. An expansion of the range \([0, p^*]\) of entrepreneurs borrowing from nonmonitoring banks.
4. A reduction in credit spreads \( R_p^* - R_0^* \).
5. An reduction in the monitoring intensity \( m_p^* \) and hence an increase in the probability of failure \( p - m_p^* \) of monitoring banks.

We illustrate these results for the case where the monitoring cost function is quadratic and the relationship between the success return \( R_p \) and the aggregate investment \( x_p \) of entrepreneurs of type \( p \) is given by (13). In this case, solving the first-order condition (19) we obtain the following equilibrium monitoring intensity
\[ m_p^* = p - \left( 1 - \sqrt{\frac{R_0^*}{\gamma}} \right) = p - p^*, \text{ for } p > p^*. \]
This implies \( p - m_p^* = p^* \), so all monitoring banks have the same probability of failure, which equals the type \( p^* \) of the marginal entrepreneur. Thus, in this case \( p^* \) fully characterizes the risk of the banking system.

Substituting this result in (18) gives the following equilibrium loan rates for types \( p > p^* \)
\[ R_p^* = \gamma(p - p^*) + \frac{R_0^*}{1 - p^*}. \]
Thus, equilibrium loan rates $R_p^*$ and credit spreads $R_p^{**} - R_0^*$ for monitoring banks are linear in the risk type $p$.

Figure 2 shows the effects of an increase in the aggregate supply of savings $w$.$^{12}$ Equilibrium variables before the change are indicated with a star and represented by solid lines, while equilibrium variables after the change are indicated with two stars and represented by dashed lines. The horizontal axis of the four panels represents entrepreneurs’ types $p$. All panels show the shift in the position of the marginal type from $p^*$ to $p^{**}$. To explain this result notice that the reduction in interest rate spreads associated with the increase in $w$ implies that banks lending to entrepreneurs of types slightly above $p^*$ will have an incentive to reduce their monitoring. But since $m_p^*$ is close to zero they will move to a corner solution with $m_p^{**} = 0$, so the nonmonitoring region will expand.

Panel A shows the effect on equilibrium loan rates. The increase in $w$ shifts downwards the function $R_p^*$ to $R_p^{**}$. The intercept of these functions is the interest rate charged to entrepreneurs of type $p = 0$ (the safe rate), which goes down from $R_0^*$ to $R_0^{**}$. To the left of the marginal types $p^*$ and $p^{**}$, loan rates are convex in $p$ (and given by $R_p^*/(1 - p)$), while to the right of these points they are linear (and given by (23)).

Panel B shows the effect on equilibrium investment allocations. The increase in $w$ shifts upwards the function $x_p^*$ to $x_p^{**}$. The total amount of lending by nonmonitoring banks is clearly increasing, since banks in the region $[0, p^*]$ will increase their lending, and banks in the region $(p^*, p^{**}]$ will switch from monitoring to not monitoring their borrowers. The effect on the total amount of lending by monitoring banks is in principle ambiguous, because fewer banks monitor their borrowers although each one increases its lending.

Panel C shows the effects on equilibrium spreads. As stated in Proposition 5, credit spreads go down from $R_p^{**} - R_0^*$ to $R_p^{**} - R_0^{**}$. Since equilibrium loan rates for monitoring banks are linear in $p$ with a slope equal to $\gamma$ (see (23)), it follows that for types riskier than $p^{**}$ spreads will be reduced by a constant amount.

$^{12}$We use $\gamma = 2$ in the monitoring cost function (2) and $\sigma = 2$ in the inverse loan demand function (13).
Finally, Panel D shows the effect on equilibrium probabilities of bank failure. The shift of entrepreneurs with types in the interval between $p^*$ and $p^{**}$ from monitoring to nonmonitoring banks means that their probability of default will go up. Also, banks that monitor will increase their probability of failure from $p - m_p^* = p^*$ to $p - m_p^{**} = p^{**} > p^*$. Thus, the increase in the aggregate supply of savings $w$ has an extensive margin effect due to the shift of nonmonitoring banks toward riskier entrepreneurs (shown by the horizontal arrow), and an intensive margin effect due to the reduction in the intensity of monitoring by monitoring banks (shown by the vertical arrow). Hence, we conclude that an increase in the supply of
savings increases the risk of the banking system.

These results provide a consistent explanation of a number of stylized facts of the period preceding the 2007-2009 financial crisis; see, for example, Brunnermeier (2009). They also provide an explanation of the way in which changes leading to a reduction in the equilibrium real rate of interest, as those noted by Summers (2014), can be linked to an increase in financial instability. In particular, one can show that an exogenous increase in the supply of savings \( w \) have the same effects on loan rates, credit spreads, and bank risk as an exogenous decrease in the demand for investment, which may be captured by a decrease of parameter \( a \) of the function \( R_p \) in (13). To see this, simply substitute (13) into the market clearing condition (21), which gives

\[
\int_0^1 x_p^* \ dp = a \int_0^1 (R_p^*)^{-\sigma} \ dp = w.
\]

Clearly, equilibrium allocations only depend on the ratio \( w/a \), so we conclude that the effects of an increase in the supply of savings are identical to the effects of fall in the demand for investment.

### 3.2 Efficiency of equilibrium

We now address whether the equilibrium of the model is constrained efficient, that is whether a social planner subject to the same moral hazard problem as the banks could improve upon the competitive equilibrium allocation. We show that the equilibrium allocation is constrained inefficient: The social planner would shift investments toward safer entrepreneurs, which will widen credit spreads and increase monitoring, thereby ameliorating the moral hazard problem.

To characterize the constrained efficient allocation we have to derive the objective function of the social planner. This requires computing the social surplus \( S_p \) associated with output \( x_p \) of entrepreneurs of type \( p \) (which obtains with probability \( 1 - p + m_p \)). To do this we use our previous derivation of the function \( R(x_p) \) in (13) from either the demand of a set of final good producers that use entrepreneurs’ output as an intermediate input, or from the demand of a representative consumer with a utility function over the continuum of goods
produced by entrepreneurs of different types.

In the case where \( R(x_p) \) is derived from the demand of a set of final good producers we have

\[
S_p = \int_{R_p}^{\infty} (\theta_p - R_p) g(\theta_p) d\theta_p + (R_p - B_p) x_p + B_p x_p,
\]

where the first term are the profits of the final good producers, the second term the profits of the banks, and the third the revenues of investors. Then using (12) and the assumption \( g(\theta_p) = a\sigma (\theta_p)^{-(\sigma+1)} \) this simplifies to

\[
S_p = \frac{a\sigma}{\sigma - 1} \left( \frac{x_p}{a} \right)^{\frac{\sigma - 1}{\sigma}}.
\]

(24)

In the case where \( R(x_p) \) is derived from the demand of a representative consumer we have

\[
S_p = \left[ \frac{a\sigma}{\sigma - 1} \left( \frac{x_p}{a} \right)^{\frac{\sigma - 1}{\sigma}} - R_p x_p \right] + (R_p - B_p) x_p + B_p x_p,
\]

where the first term is the surplus of the representative consumer, the second term the profits of the banks, and the third the revenues of investors, which also gives (24).

A \textit{constrained efficient allocation} \( \{\bar{x}_p\}_{p \in [0,1]} \) is an allocation that maximizes expected social surplus net of monitoring costs

\[
\int_0^1 [(1 - p + m_p) S_p - c(m_p) x_p] \, dp,
\]

subject to the condition that characterizes optimal monitoring under moral hazard

\[
m_p = \max \left\{ m \in [0, p] \mid c'(m) + \frac{\bar{R}_0}{1 - p + m} \leq \bar{R}_p \right\}, \text{ for all } p \in [0, 1],
\]

(25)

and the market clearing condition

\[
\int_0^1 \bar{x}_p \, dp = w
\]

(26)

where \( \bar{R}_p = R(\bar{x}_p) \) for all \( p \in [0, 1] \).

Since the social planner is subject to the same moral hazard problem as the banks, condition (25) follows from the characterization of the optimal contract between the bank and the investors in Proposition 1. Condition (26) ensures that the aggregate demand for investment is equal to the aggregate supply of savings \( w \).
For those types $p$ for which $m_p = 0$, differentiating the Lagrangian with respect to $x_p$ and using the expressions for $R_p$ and $S_p$ in (13) and (24) gives

$$(1 - p)R_p = \lambda,$$  
(27)

where $\lambda$ is the Lagrange multiplier associated with the market clearing condition (26). For the type $p = 0$ this implies $R_0 = \lambda$, so the Lagrange multiplier is the safe interest rate.

For those types $p$ for which $m_p > 0$, differentiating the Lagrangian with respect to $x_p$ and using the expressions for $R_p$ and $S_p$ in (13) and (24) gives

$$(1 - p + m_p)R_p - c(m_p) + \left[ \frac{\sigma}{\sigma - 1}R_p - c'(m_p) \right] x_p \frac{dm_p}{dR_p} \frac{dR_p}{dx_p} = \lambda.$$  
(28)

From here one can show that, whenever $m_p > 0$, the constrained efficient allocation is characterized by loan rates $R_p$ higher than the lowest feasible rate $R_p$ that characterizes the competitive equilibrium allocation. To see this, notice that if the monitoring intensity $m_p$ satisfied the first-order condition (19) that defines $R_p$, then we would have $dm_p/dR_p = \infty$. In such case (28) could not hold.\textsuperscript{13} Thus, in the monitoring region the social planner restricts investment in order to widen credit spreads and increase monitoring.

These results are in line with the intuition of the traditional literature on the relationship between competition and risk-taking in banking; see, for example, Repullo (2004). This literature shows that higher market power leads to higher intermediation margins (spreads in our model) and lower probabilities of bank failure.

Figure 3 illustrates a special case in which the constrained efficient allocation entails full monitoring for all types of entrepreneurs, that is $m_p = p$ for all $p$.\textsuperscript{14} Variables corresponding to the competitive equilibrium allocation are indicated with a star and represented by solid lines, while variables corresponding to the constrained efficient allocation are indicated with two stars and represented by dashed lines. The horizontal axis of the four panels represents entrepreneurs’ types. All panels show the shift in the position of the marginal type from $p^*$ to $p^{**} = 0$. Panel B shows that, compared to the equilibrium allocation, the social planner

\textsuperscript{13}Notice that by (25) the term in square brackets in (28) is positive.

\textsuperscript{14}Functional forms and parameter values are the same as those in Figure 2, except for parameter $\gamma$ of the monitoring cost function which is increased from 2 to 4 to make the effects more visible.
This figure shows loan rates (Panel A), investment (Panel B), spreads (Panel C), and the probability of failure (Panel D) for the competitive equilibrium allocation (solid lines) and the constrained efficient allocation (dashed lines).

reallocates investments toward safer entrepreneurs. This reallocation results in an increase in loan rates for riskier entrepreneurs and a reduction in rates for safer entrepreneurs, as illustrated in Panel A. Consequently, credit spreads go up, as illustrated in Panel C, which leads to the reduction in the probabilities of bank failure to zero shown in Panel D.\footnote{The corner solution in the probabilities of bank failure is a special case. For higher monitoring costs some banks would set $m_p < p$.}
4 Endogenous Booms and Busts

We have so far analyzed the equilibrium of a static model for a given aggregate supply of savings and shown how an exogenous change in this supply affects interest rates, credit spreads, and the structure and risk of the banking system. This section analyzes a dynamic extension of the static model in which investors are infinitely lived and the aggregate supply of savings is endogenous. Specifically, the supply of savings at any date is the outcome of investors’ consumption and savings decisions at the previous date, together with the realization of a systematic risk factor that affects the return of entrepreneurs’ projects.

The dynamic model generates endogenous booms and busts: The accumulation of savings leads to a reduction in interest rates and credit spreads, which result in an increase in risk-taking that makes a bust more likely to occur. The bust reduces savings, increasing interest rates and credit spreads, starting again the process of wealth accumulation that leads to a boom.

At each date $t$, there is a continuum of one-period-lived penniless entrepreneurs of types $p \in [0,1]$ that have unit-sized investment projects which can only be funded by banks. As before, we assume that the banking sector is contestable and that there is a single bank that lends to entrepreneurs of type $p$ at date $t$, choosing the monitoring intensity $m_{pt} \in [0,p]$. The project of an entrepreneur of type $p$ yields at date $t+1$ a return $R_{pt} = R(x_{pt})$ with probability $p$ and zero with probability $1-p+m_{pt}$, where $x_{pt}$ denotes the aggregate investment of entrepreneurs of type $p$ at date $t$, and $R(x)$ is given by (13).

At each date $t$, there is a continuum of measure $w_t$ of infinitely-lived risk-neutral atomistic investors with unit wealth. Investors have a discount factor $\beta \in (0,1)$ and the period utility function is $u(c_t) = c_t$. These investors fund the banks which in turn fund entrepreneurs’ projects. To simplify the presentation, we assume that banks are run by penniless one-period-lived bankers that consume the profits that they obtain before they die. Thus, banks have no inside equity capital, and bank profits do not contribute to the accumulation of wealth.

To describe the realization of project returns, we maintain the assumption that the
returns of the projects of entrepreneurs of each type $p$ are perfectly correlated, but we assume that project returns are imperfectly correlated across types. Specifically, we will use the \textit{single risk factor model} of Vasicek (2002) in which the outcome of the projects of entrepreneurs of type $p$ is driven by the realization of a latent random variable

$$y_{pt} = -\Phi^{-1}(p - m_{pt}) + \sqrt{\rho} z_t + \sqrt{1 - \rho} \varepsilon_{pt},$$

where $z_t$ is a \textit{systematic risk factor} that affects all types of entrepreneurs, $\varepsilon_{pt}$ is an idiosyncratic risk factor that only affects the projects of entrepreneurs of type $p$. It is assumed that $z_t$ and $\varepsilon_{pt}$ are standard normal random variables, independently distributed from each other and over time as well as, in the case of $\varepsilon_{pt}$, across types. Parameter $\rho \in (0, 1)$ determines the extent of correlation in the returns of the projects of entrepreneurs of different types, $\Phi(\cdot)$ denotes the cdf of a standard normal random variable, and $\Phi^{-1}(\cdot)$ its inverse.

The projects of entrepreneurs of type $p$ fail at date $t$ when $y_{pt} < 0$. Hence, their probability of failure is

$$\Pr(y_{pt} < 0) = \Pr \left[ \sqrt{\rho} z_t + \sqrt{1 - \rho} \varepsilon_{pt} < \Phi^{-1}(p - m_{pt}) \right] = p - m_{pt}.$$  

By our assumptions, the dynamic behavior of aggregate wealth can be expressed as

$$w_{t+1} = G(w_t, z_t) = \int_0^1 \delta(p - m_{pt}, z_t)x_{pt}B_{pt} \, dp,$$  

where

$$\delta(p - m_{pt}, z_t) = \Pr(y_{pt} \geq 0 \mid z_t) = \Phi \left( \frac{\sqrt{\rho} z_t - \Phi^{-1}(p - m_{pt})}{\sqrt{1 - \rho}} \right).$$  

The integrand in the right-hand side of (30) is the conditional (on the realization of the systematic risk factor $z_t$) probability of success of the projects of entrepreneurs of type $p$ at date $t$ multiplied by the payment to investors in case of success. Such payment is equal to the product of the investment $x_{pt}$ by the interest rate $B_{pt}$ at which they lend to the corresponding bank. Since the systematic risk factor $z_t$ is a random variable, the dynamic behavior of aggregate wealth is also random.

For expository purposes we assume that investors can either consume their unit wealth, invest it in the bank lending to entrepreneurs of type $p = 0$, or invest it in the bank lending
to entrepreneurs of an arbitrary type \( p > 0 \).\(^{16}\) Let \( s_0 \in [0, 1] \) and \( s_p \in [0, 1] \) denote the amounts invested in the two banks, and \( c = 1 - s_0 - s_p \in [0, 1] \) the amount consumed. The Bellman equation is then given by

\[
v(w_t) = \max_{(s_0, s_p)} \{1 - s_0 - s_p + \beta [s_0 R_{0t} E[v(w_{t+1})] + s_p B_{pt} E[\delta(p - m_{pt}, z_t)v(w_{t+1})]]\}.
\]  

(32)

Assuming that \( \lim_{x \to 0} R(x) = \infty \), as in (13), in equilibrium we must have \( s_0 > 0 \) and \( s_p > 0 \). Then, differentiating the right-hand side of (32) with respect to \( s_0 \) and \( s_1 \), equating to zero the resulting expressions, and subtracting one from the other, gives the following condition

\[
R_{0t} E[v(w_{t+1})] = B_{pt} E[\delta(p - m_{pt}, z_t)v(w_{t+1})].
\]

(33)

This condition states that investors must be indifferent between lending to the two banks.

In equilibrium it must be the case that \( v(w) \geq 1 \), since investors can always set \( s_0 = s_p = 0 \), consuming all their wealth, which gives \( u(1) = 1 \). It must also be the case that \( v(w) > 1 \) only if \( c = 1 - s_0 - s_p = 0 \), since if \( c > 0 \) the marginal utility of lending to any of the two banks must be equal to the marginal utility of consumption which is one.

Let us now define

\[
\hat{w} = \inf \{w \mid v(w) = 1\}.
\]

(34)

Clearly, we have \( v(w) = 1 \) for all \( w \geq \hat{w} \). Thus, when \( w < \hat{w} \) the value of one unit of wealth is greater than one and investors do not consume, while when \( w \geq \hat{w} \) the value of one unit of wealth is equal to one and they invest \( \hat{w} \) and devote the difference \( w - \hat{w} \) to consumption. Hence, the aggregate consumption of investors is given by

\[
c(w_t) = \begin{cases} 
    w_t - \hat{w}, & \text{for } w_t \geq \hat{w}; \\
    0, & \text{for } w_t < \hat{w}.
\end{cases}
\]

(35)

Substituting the indifference condition (33) into the Bellman equation (32) gives

\[
v(w_t) = \beta R_{0t} E[v(w_{t+1})] = \beta B_{pt} E[\delta(p - m_{pt}, z_t)v(w_{t+1})],
\]

(36)

\(^{16}\)As will be clear below, focussing on these two banks is without loss of generality, as investors will be indifferent among any of the banks.
which implies the fundamental pricing equation

\[ E \left[ \frac{\beta v(w_{t+1})}{v(w_t)} \delta(p - m_{pt}, z_t) B_{pt} \right] = 1, \quad (37) \]

where \( \beta v(w_{t+1})/v(w_t) \) is the stochastic discount factor.

Given that the expected value of the stochastic discount factor equals the inverse of safe rate \( R_{0t} \), and that \( E \left[ \delta(p - m_{pt}, z_t) \right] = \Pr(y_{pt} \geq 0) = 1 - p + m_{pt} \), we have

\[ E \left[ \frac{\beta v(w_{t+1})}{v(w_t)} \delta(p - m_{pt}, z_t) B_{pt} \right] = \frac{(1 - p + m_{pt})B_{pt}}{R_{0t}} + Cov \left[ \frac{\beta v(w_{t+1})}{v(w_t)}, \delta(p - m_{pt}, z_t) B_{pt} \right], \]

so the pricing equation (37) implies

\[ (1 - p + m_{pt})B_{pt} - R_{0t} = -R_{0t} Cov \left[ \frac{\beta v(w_{t+1})}{v(w_t)}, \delta(p - m_{pt}, z_t) B_{pt} \right]. \]

Since \( w_{t+1} = G(w_t, z_t) \) and \( \delta(p - m_{pt}, z_t) \) are both increasing in the systematic risk factor \( z_t \), and \( v'(w_{t+1}) \leq 0 \), with strict inequality for \( w_{t+1} < \hat{w} \), we conclude that the covariance term is negative. This implies

\[ (1 - p + m_{pt})B_{pt} - R_{0t} > 0, \text{ for all } p > 0. \]

Thus, \textit{investors require positive risk premia for funding the risky banks.}\(^{17}\)

Following the same steps as in the analysis of the static model in Section 3, and solving for \( B_{pt} \) in (33), one can show that banks lending to entrepreneurs of types \( p > 0 \) set the lowest feasible loan rate, which is given by

\[ R_{pt} = \min_{m_{pt} \in [0, p]} \left( c'(m_{pt}) + \frac{R_{0t} E[v(w_{t+1})]}{E[\delta(p - m_{pt}, z_t)v(w_{t+1})]} \right). \quad (38) \]

Notice that in the static model we have \( v(w_{t+1}) = 1 \), which implies \( E[\delta(p - m_{pt}, z_t)v(w_{t+1})] = 1 - p + m_{pt} \). Hence, \( R_{pt} \) in (15) is a special case of (38).

### 4.1 Equilibrium of the dynamic model

We are now ready to define an equilibrium, which requires to specify the investment \( x_{pt} \) of the different types of entrepreneurs, and hence the rates \( R_{pt} = R(x_{pt}) \) at which they will borrow

\(^{17}\)This means that the behavior of investors in the dynamic model is similar to their behavior in the static model with risk-averse investors analyzed in Appendix A.
from banks, the rates $B_{pt}$ at which banks will borrow from investors, and their monitoring intensity $m_{pt}$. All these variables depend on the wealth $w_t$ of investors, which is the state variable of the dynamic model. The equilibrium also requires to specify the value function of investors $v(w_t)$, their aggregate consumption decision $c(w_t)$, and the dynamics of wealth accumulation described by $w_{t+1} = G(w_t, z_t)$.

Formally, an equilibrium is an array $\{x^*_p(w_t), R^*_p(w_t), B^*_p(w_t), m^*_p(w_t)\}_{p \in [0,1]}; v(w_t), c(w_t), G(w_t, z_t)$ such that

1. Entrepreneurs’ investment decisions satisfy $R^*_p(w_t) = R\left(x^*_p(w_t)\right)$,
2. Banks’ lending rates $R^*_p(w_t)$ equal the lowest feasible rates $\overline{R}_{pt}$ in (38),
3. Banks’ borrowing rates $B^*_p(w_t)$ satisfy the fundamental pricing equation (37),
4. Banks’ monitoring intensity $m^*_p(w_t)$ solves (38),
5. The value function $v(w_t)$ satisfies (36),
6. The consumption function $c(w_t)$ satisfies (35),
7. The investors’ wealth $w_t$ evolves according to (30), and
8. The market clears

$$\int_0^1 x^*_p(w_t) \, dp = w_t - c(w_t).$$

We can analyze the equilibrium of the dynamic model using the parameterization in Section 3. As in the static setup, there is a marginal type $p^*_t$ such that banks lending to entrepreneurs of types $p \leq p^*_t$ do not monitor their borrowers, and banks lending to types $p > p^*_t$ will monitor them, setting $m^*_{pt} = p - p^*_t$. This implies $p - m^*_{pt} = p^*_t$, so all monitoring banks have the same probability of failure, which equals the type $p^*_t$ of the marginal entrepreneur.\textsuperscript{18}

\textsuperscript{18}To prove this result, suppose that the solution to (38) for some $p$ is such that $m^*_{pt} > 0$. Then, $m^*_{pt}$
The indifference condition (33) and the condition (38) that characterizes equilibrium loan rates imply

\[ R_{pt}^* = \gamma m_{pt}^* + B_{pt}^*. \]

Hence, banks’ lending and borrowing rates, \( R_{pt}^* \) and \( B_{pt}^* \), coincide for \( p \leq p_t^* \), that is for nonmonitoring banks, and satisfy \( R_{pt}^* > B_{pt}^* \) for \( p > p_t^* \), that is for monitoring banks. In this case, as discussed in Section 3, the intermediation margin \( R_{pt}^* - B_{pt}^* \) covers (in expected terms) the monitoring cost and leaves some positive profits for banks.

Figure 4 illustrates the value function \( v(w_t) \) of the dynamic model, and the marginal type \( p^* \) (for different values of wealth \( w \)) in the static and the dynamic model.\(^{19}\) The value function in Panel A is decreasing and convex for \( w < \hat{w} \), and satisfies \( v(w) = 1 \) for \( w \geq \hat{w} \). This condition is also satisfied for any \( p' \) \( \geq p_t^* \) with \( m_{p't}^* = p' - p_t^* \).

\(^{19}\) We assume a discount factor \( \beta = 0.96 \) and a correlation parameter \( \rho = 0.15 \).
As noted above, the fact that \( v'(w) \leq 0 \), with strict inequality for low values of \( w \), implies the existence of positive risk premia. The positive risk premia opens up credit spreads and increases the value of monitoring. This is illustrated in Panel B, which shows that the marginal type in the dynamic model (solid line) is everywhere below the marginal type of the static model (dashed line), except for low values of wealth when \( p^* = 0 \) in both models. This means that, for any given level of wealth, the dynamic model features a smaller relative size of the nonmonitoring banking system, and a lower probability of failure of the monitoring banks. In other words, the forward-looking behavior of investors contributes to the stability of the banking system.

The dynamic model yields a number of potentially testable relationships between aggregate variables. In order to highlight some of these relationships, in what follows we consider a sample realization of the (iid) systematic risk factor \( z_t \) and look at the corresponding evolution of investors’ wealth \( w_t \) over time together some interesting variables. The dashed line of Panels A-D of Figure 5 represents \( w_t \), and the horizontal thin line shows the value \( \hat{w} \) above which investors consume \( w_t - \hat{w} \).

In Panel A the solid line plots the total amount of lending by monitoring banks, that is \( \int_{p_t}^{1} x_p^*(w_t) \, dp \), and the dotted line plots the total amount of lending by nonmonitoring banks, that is \( \int_{0}^{p_t} x_p^*(w_t) \, dp \). Although lending by both types of banks is positively correlated with investors’ wealth, most of the variation in \( w_t \) is channeled through nonmonitoring banks. This result that is consistent with the evidence in Pozsar, Adrian, Ashcraft, and Boesky (2012) that shows that shadow bank liabilities grew much faster than traditional banking liabilities in the run-up to the crisis, and contracted substantially following the 2007 peak.

The solid line in Panel B show the evolution of the risk premia \( (1 - p + m_{pt}^*)B_{pt}^* - R_{0t}^* \) for a type \( p \) that is always funded by nonmonitoring banks. The fact that risk premia go down when wealth accumulates means that the dynamic model yields *countercyclical risk premia*. Thus, investors behave as if they were less risk-averse (or have greater risk appetite) during financial booms, although their underlying (risk-neutral) preferences do not change.
Figure 5. Model dynamics

The dashed line in all panels shows the dynamic evolution of investors’ wealth for a sample realization of the systematic risk factor, and the horizontal thin line shows the threshold level of wealth above which investors consume. Panel A plots the total amount of lending by monitoring banks (solid line) and nonmonitoring banks (dotted line). The solid lines in Panels B, C, and D represent the risk premia for investors in a given nonmonitoring bank, the probability of failure of monitoring banks, and the safe interest rate.

The solid line in Panel C shows the evolution of the type $p_t^*$ of the marginal entrepreneur. As illustrated in Panel B of Figure 4, this variable is positively correlated with $w_t$. Thus, in line with the results of the static model, higher wealth increases the relative size of the nonmonitoring banking system, and increases the probability of failure of the monitoring banks.

Finally, the solid line in Panel D shows the evolution of the safe interest rate $R_{0t}$. This variable is negatively correlated with $w_t$. It is also interesting to note that the safe rate can
be below the discount rate $1/\beta$.\textsuperscript{20} The intuition is that the expectation of positive returns in the future (when the economy is hit by a negative shock and wealth is very valuable) makes investors willing to forgo current consumption, which lowers the safe rate.

5 Conclusion

This paper presents a general equilibrium model of the connection between real interest rates, credit spreads, and the structure and risk of the banking system. Banks intermediate between a heterogeneous set of entrepreneurs characterized by their risk type, and a set of investors characterized by their aggregate supply of savings. We assume that all agents are risk-neutral and that banks can decide the monitoring intensity of entrepreneurs' projects at a cost, but this is not observed by investors. This moral hazard problem is the key friction that drives the results of the model. We also assume that project returns are decreasing in the aggregate investment of entrepreneurs of each type, and that the market for lending to each type of entrepreneurs is contestable.

We first characterize the competitive equilibrium of the model, showing that safer entrepreneurs are funded by banks that do not monitor their projects and riskier entrepreneurs by banks that monitor them. Assuming that monitoring requires keeping the loans in the banks' books, we associate monitoring banks with traditional banks that originate-to-hold, and nonmonitoring banks with direct market finance or with intermediaries that originate-to-distribute (a part of the shadow banking system).

We then analyze the effects of an exogenous increase in the supply of savings, showing that it will lead to a reduction in interest rates and credit spreads, an expansion of the relative size of the nonmonitoring banking system, and a reduction in the monitoring intensity and hence an increase in the probability of failure of monitoring banks. The competitive equilibrium is shown to be constrained inefficient. A social planner subject to the same moral hazard problem as the banks would shift investments from riskier to safer entrepreneurs in order to widen credit spreads and thereby increase monitoring incentives.

\textsuperscript{20} Notice that for $w_t \geq \tilde{w}$ we have $v(w_t) = 1$ and $E[v(w_{t+1})] > 1$, so (36) implies $\beta R_{0t} < 1$, i.e. $R_{0t} < 1/\beta$. 

30
We extend our static model to a dynamic setup in which the aggregate supply of savings at any date is the outcome of investors’ decisions at the previous date together with the realization of a systematic risk factor that affects the success of entrepreneurs’ projects. The dynamic model generates endogenous booms and busts cycles. The accumulation of savings in a boom leads to a reduction in interest rates and credit spreads, an increase in the size of the nonmonitoring banking system, and an increase in the probability of failure of monitoring banks. These changes make the economy especially vulnerable to shocks making a bust more prone to happen. The reduction in savings in a bust increases interest rates and credit spreads, starting again the process of wealth accumulation that leads to a boom. The dynamic model also yields a number of empirically relevant results such as the procyclical nature of the nonmonitoring banking system, the existence of countercyclical risk premia, and the low levels of interest rates and credit spreads leading to the buildup of risks during booms.

Our results provide a theoretical explanation for the relationship noted by Rajan (2005) between high savings, low real interest rates, and the incentives of financial intermediaries to search for yield. They also rationalize the way in which factors leading to a reduction in real interest rates, as those noted by Summers (2014), can be associated with an increase in financial instability. Moreover, the results provide a rationale for a number of empirical facts in the run-up of the 2007-2009 financial crisis.

We would like to conclude with three remarks. First, our model builds on the assumption that banks monitor their borrowers, an idea that traces back to the seminal work of Diamond (1984).\textsuperscript{21} Still, a similar story could be constructed if banks increase the quality its pool of loan applicants by screening them.\textsuperscript{22} Second, the paper entirely focuses on debt finance, abstracting from the possibility of funding the banks with (inside) equity. Since equity finance would strengthen the banks’ monitoring incentives, adding this possibility would

\textsuperscript{21}The relevance of this assumption has been recently documented by Wang and Xia (2014), who show that “banks active in securitization impose looser covenants on borrowers at origination” and “after origination, these borrowers take on substantially more risk than do borrowers of non-securitization-active banks.”

\textsuperscript{22}This setup would be in essence similar to the one in Helpman, Itskhoki and Redding (2010), where firms increase the average ability of the workers they hire by paying a screening cost.
require the introduction of a differential cost of equity. Finally, it is important to stress that we have constructed a model without any nominal frictions, so monetary policy is completely absent from our story of search for yield. Introducing nominal frictions would allow to study the connection between monetary policy and financial stability, a relevant topic that merits a separate analysis.
Appendix

A  Risk-averse investors

This Appendix extends the model of Section 3 to the case where investors are risk-averse instead of risk-neutral. This allows us to distinguish the effects of a change in the supply of savings from those of a change in investors’ risk appetite.

Specifically, suppose that there is a continuum of measure \( w \) of atomistic investors with unit wealth that can be invested in only one bank (so we do not allow any portfolio diversification). Since each investor has a unit wealth, the measure of investors \( w \) is equal to the aggregate supply of savings.

Investors have a constant relative risk aversion utility function. Given that bank assets can yield a zero return, we restrict the coefficient of relative risk aversion to be between zero and one. Thus, we have

\[
\begin{align*}
\begin{align*}
    u(c) &= c^{1/\alpha},
\end{align*}
\end{align*}
\]

where \( \alpha \geq 1 \). Risk-neutrality corresponds to the limit case \( \alpha = 1 \).

An in our risk-neutral setup, in equilibrium investors have to be indifferent between funding banks lending to different types of entrepreneurs. This implies that in the definition of an optimal contract between a bank lending to entrepreneurs of type \( p \) and the risk-averse investors, the participation constraint (6) becomes

\[
(1 - p + m_p^*) \left( B_p^* \right)^{1/\alpha} = R_0^{1/\alpha},
\]

which may be rewritten as

\[
B_p^* = \frac{R_0}{(1 - p + m_p^*)^\alpha}.
\]

Notice that this implies that investors’ expected payoff satisfies

\[
(1 - p + m_p^*) B_p^* = \frac{R_0}{(1 - p + m_p^*)^{\alpha - 1}} > R_0,
\]

so investors require positive risk premia.
Substituting the participation constraint (40) into the first-order condition (7) gives
\[ c'(m_p) + \frac{R_0}{(1 - p + m_p)^\alpha} = R. \]
(41)

Let \( R_p \) denote the minimum value of the function in left-hand side of (41). Then, we can follow the same steps as in the proof of Proposition 1 to show that bank finance is feasible if \( R \geq R_p \). In such case, the optimal contract between the bank and the risk-averse investors is
\[ m_p^* = \max \left\{ m \in [0, p] \mid c'(m) + \frac{R_0}{(1 - p + m)^\alpha} \leq R \right\}. \]

Thus, we have essentially the same characterization of the optimal contract as in the risk-neutral setup analyzed in Section 2. The difference is that the convex function in the left-hand side of (41) is increasing in \( \alpha \); so risk aversion makes it more difficult to ensure the feasibility of bank finance.

Following the same steps as in Section 3, we can characterize the equilibrium of the model with risk-averse investors. In this equilibrium, the marginal type is given by
\[ p^* = 1 - \left( \frac{\alpha R_0^*}{c''(0)} \right)^{\frac{1}{1 + \alpha}}. \]

Notice that \( p^* \) is decreasing in the safe rate \( R_0^* \) (as before) and also in the risk-aversion parameter \( \alpha \). Thus, risk-aversion increases the comparative advantage of monitoring banks.

Figure 6 shows the effect of a reduction in investors’ risk aversion from \( \alpha = 2 \) to \( \alpha = 1 \) (risk-neutrality) for the same parameterization used in Section 3. As before, equilibrium variables before the change are indicated with a star and represented by solid lines, while equilibrium variables after the change are indicated with two stars and represented by dashed lines. The horizontal axis of the four panels represents entrepreneurs’ types \( p \).

Panel A shows the effect on equilibrium loan rates. The reduction in risk aversion shifts the investors’ preferences toward riskier assets, so loan rates go down for riskier entrepreneurs and go up for safer entrepreneurs. In particular, the safe rate will move from \( R_0^* \) to \( R_0^{**} \). The increase in the safe rate together with the reduction in loan rates for riskier entrepreneurs reduces the comparative advantage of monitoring banks, which explains the shift in the position of the marginal type from \( p^* \) to \( p^{**} \).
Figure 6. Effects of a reduction in investors’ risk aversion

This figure shows the effects of a change in investors’ preferences from risk aversion to risk neutrality on equilibrium loan rates (Panel A), investment (Panel B), spreads (Panel C), and the probability of failure (Panel D) for different types of entrepreneurs. Solid (dashed) lines represent equilibrium values before (after) the change in preferences.

Panel B shows the effect on equilibrium investment allocations. The reduction in risk aversion produces a reallocation of savings toward riskier entrepreneurs. Since the aggregate supply of savings is fixed, this means that investment in safer projects falls. However, the shift in the position of the marginal type from $p^*$ to $p^{**}$ implies that the effect on the relative size of the nonmonitoring banking system is ambiguous.

Panel C shows the effects on equilibrium spreads. The results on loan rates imply that credit spreads go down from $R_p - R_0$ to $R_p^{**} - R_0^{**}$. This reduces the incentives to monitor and hence increases the probability of failure of monitoring banks, which is shown in Panel
D. As in the case of the increase in the supply of savings, a reduction in risk-aversion has an extensive margin effect (shown by the horizontal arrow), and an intensive margin effect (shown by the vertical arrow).

These results illustrate the differences between the effects of a savings glut from the effects of a reduction in the investors’ risk appetite. Both changes lead to a reduction in credit spreads and an increase in the probability of failure of monitoring banks, but there are some significant differences. A savings glut increases funding for all types of entrepreneurs and total lending by nonmonitoring banks, while a fall in risk aversion reduces funding for safer entrepreneurs and has an ambiguous effect on total lending by nonmonitoring banks. A simple way to empirically distinguish the two changes is to look at the effect on the equilibrium safe rate: it goes down in the case of a savings glut and it goes up in the case of a reduction in risk aversion.

B Proofs

Proof of Proposition 1 If \( R < R \), for any \( m \in (0, p] \) we have

\[
R - \frac{R_0}{1 - p + m} - c'(m) < 0,
\]

which implies that the bank has an incentive to reduce \( m \). But for \( m = 0 \) we have

\[
R - \frac{R_0}{1 - p} - c'(0) < 0.
\]

Using the assumption \( c(0) = c'(0) = 0 \), this implies \((1 - p)R - R_0 - c(0) < 0\), which violates the bank’s participation constraint (5).

If \( R \geq R \), by the convexity of the function in the left-hand side of (8) there exist an interval \([m^-, m^*] \subset [0, p] \) such that

\[
R - \frac{R_0}{1 - p + m} - c'(m) \geq 0 \quad \text{if and only if} \quad m \in [m^-, m^*].
\]

By our previous argument, for any \( m \in (0, p] \) for which

\[
R - \frac{R_0}{1 - p + m} - c'(m) < 0,
\]
the bank has an incentive to reduce \( m \). Similarly, for any \( m \in [0, p) \) for which
\[
R - \frac{R_0}{1 - p + m} - c'(m) > 0,
\]
the bank has an incentive to increase \( m \). Hence, there are three possible values of monitoring in the optimal contract: \( m = m^* \), \( m = m^- \), and \( m = 0 \) (when \( m^- > 0 \)).

To prove that the bank prefers \( m = m^* \), notice that our assumptions on the monitoring cost function together with the definition of \( m^* \) imply
\[
\frac{d}{dm} [(1 - p + m)R - c(m)] = R - c'(m) > R - c'(m^*) \geq B^* > 0,
\]
for \( m < m^* \). Hence, we have
\[
(1 - p + m^*)(R - B^*) - c(m^*) = (1 - p + m^*)R - R_0 - c(m^*)
\]
\[
> (1 - p + m)R - R_0 - c(m) = (1 - p + m)(R - B) - c(m),
\]
for either \( m = m^- \) or \( m = 0 \) (when \( m^- > 0 \)).

Finally, to prove that the bank’s participation constraint (5) is satisfied notice that
\[
(1 - p + m^*)R - R_0 - c(m^*) \geq (1 - p + m^*)c'(m^*) - c(m^*) > (1 - p)c'(m^*) > 0,
\]
where we have used the definition of \( m^* \) and the fact that \( m^*c'(m^*) > c(m^*) \) by the convexity of the monitoring cost function. \( \square \)

**Proof of Proposition 2** Differentiating condition (8) for an interior level of monitoring \( m^* \in (0, p) \) gives
\[
\left( c''(m^*) - \frac{R_0}{(1 - p + m^*)^2} \right) dm^* + \frac{1}{1 - p + m^*} dR_0 - dR = 0.
\]
By Proposition 1, when \( R > R \) and \( m^* < p \) the term that multiplies \( dm^* \) is positive, which implies
\[
\frac{\partial m^*}{\partial R_0} = -\frac{1}{1 - p + m^*} \left( c''(m^*) - \frac{R_0}{(1 - p + m^*)^2} \right)^{-1} < 0,
\]
\[
\frac{\partial m^*}{\partial R} = \left( c''(m^*) - \frac{R_0}{(1 - p + m^*)^2} \right)^{-1} > 0. \quad \square
\]
Proof of Proposition 3 To prove that higher types \( p \) will be characterized by higher spreads \( R_p - R_0 \) for \( p \leq \hat{p} \) note that (17) implies

\[
\frac{d(R_p - R_0)}{dp} = \frac{R_0}{(1-p)^2} > 0,
\]

and for \( p > \hat{p} \) apply the envelope theorem to (18), which gives

\[
\frac{d(R_p - R_0)}{dp} = \frac{1}{(1 - p + m_p)^2} > 0.
\]

To prove that higher types \( p > \hat{p} \) will be characterized by higher monitoring \( m_p \) we differentiate the first-order condition (19) and use the assumption \( c'''(m) \geq 0. \)

Proof of Proposition 4 To prove that increases in the safe rate \( R_0 \) lead to increases in the spreads \( R_p - R_0 \) for \( p \leq \hat{p} \) note that (17) implies

\[
\frac{d(R_p - R_0)}{dR_0} = \frac{p}{1 - p} > 0,
\]

so spreads are linear in \( R_0 \), and for \( p > \hat{p} \) apply the envelope theorem to (18), which gives

\[
\frac{d(R_p - R_0)}{dR_0} = \frac{p - m_p}{1 - p + m_p} > 0.
\]

To prove that monitoring \( m_p \) is increasing in the safe rate \( R_0 \) for \( p > \hat{p} \) we differentiate the first-order condition (19) and use the assumption \( c'''(m) \geq 0. \)
References


